Internship: Delay Differential Equations and Neural Dynamics in Machine Learning

Keywords: Machine Learning, Neural ODEs, Delayed differential equations, Dynamical Systems
Duration, salary, starting date: max. 6 months, min. 600 € per month, starting as soon as possible.
Research teams: DISCO (Inria, CNRS, CentraleSupélec, Université Paris-Saclay), MLIA (Sorbonne University and CNRS)
Advisers: Pr. Aditya Mahajan (McGill Univ., Montréal, Canada), Dr. Silviu-Iulian Niculescu (CNRS, L2S & DISCO)
Co-Adviser: Dr. Edouard Oyallon (CNRS, Sorbonne University)
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Application: Interested candidate should send CV, grade transcript and potentially one or two referees who could provide a recommendation letter.

Context: The integration of delayed differential equation (DDE) and neural ODEs in machine learning offers new perspectives for modeling complex dynamical systems [3]. A DDE delayed by τ (a scalar, or a regular function) is typically expressed, for some regular f via

$$\frac{d}{dt}x_t^\tau = f(x_{t-\tau}^\tau)\,. \tag{1}$$

This internship aims to explore the intersection of these fields, focusing on the development and analysis of advanced algorithms for delayed differential equations within neural network architectures. Three types of concepts could be explored:

- Learning new Lyapunov [2] $V_\theta(\{x_t\}_t)$ to explore the potential of using Neural Networks, where θ could be learned by minimizing
$$\inf_\theta \mathbb{E}_{\tau,x_0^\tau}[V_\theta(\{x_t^\tau\},\tau)]\,,$$
under the constraint that V_θ is a Lyapunov function (e.g., $V_\theta \geq 0$ and $\frac{d}{dt}V(x_t) \leq 0$ and τ is a L^1 random variable. This could be done via, e.g., stochastic gradient descent,
- Sampling $\tau \in \mathcal{C}^\infty$ or less regular space, again, to study the intriguing properties [] of ODEs and their stability,
- Integrating delayed Neural ODEs [3] using generative flows, in the perspective to learn, e.g., the discretized scheme:
$$x_{t+1} = x_t - F_{t,\theta,\tau}(x_{t-\tau})\,,$$
or more complex schemes likes Neural ODEs []. The goal would be to generalize over higher delays that the one on which $\theta \to F_{t,\theta,\tau}$ would have been trained and to benefit on the literature on flows [1].

Proposed work: The intern will engage in the following activities:

1. Literature review on DDEs and their applications in machine learning and the development of neural network architectures integrating DDEs for specific case studies.
2. Implementation and testing of these models using Python and PyTorch, with a focus on efficiency and scalability, as well as the analysis of the models' performance in comparison with the existing mathematical frameworks.
3. Contribution to research papers and reports detailing the findings and potential applications.

An ideal candidate should:

- Be at the Master 2 level (or equivalent),
- Have a strong background in applied mathematics and dynamical systems,
- Possess solid programming skills in Python and experience with Pytorch,
- Show interest in machine learning and its theoretical aspects.

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References

[1] Marin Biloš, Johanna Sommer, Syama Sundar Rangapuram, Tim Januschowski, and Stephan Günnemann. Neural flows: Efficient alternative to neural odes. *Advances in neural information processing systems*, 34:21325–21337, 2021.

[2] Hongkai Dai, Benoit Landry, Lujie Yang, Marco Pavone, and Russ Tedrake. Lyapunov-stable neural-network control. *arXiv preprint arXiv:2109.14152*, 2021.

[3] Qunxi Zhu, Yao Guo, and Wei Lin. Neural delay differential equations. *arXiv preprint arXiv:2102.10801*, 2021.