Understanding Deep Representations

- Separation, contraction are necessary properties of a successful model:
  \[ R^D \quad D \gg d \quad R^d \]
- How can we relate it to the depth? How can we design the non-linearity? How to measure the dimensionality reduction?

Simplified CNN framework

- Depends only on the width \( K \) and non-linearity \( \rho \)
- No max-pooling or ad-hoc non-linear modules
- \( W_j \): convolution with \( K \) inputs and outputs
- Only 13 layers!
- BatchNorm

Benchmarking

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x24</td>
<td>3x24</td>
</tr>
<tr>
<td>5x48</td>
<td>5x48</td>
</tr>
<tr>
<td>7x96</td>
<td>7x96</td>
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</tbody>
</table>

Influence of the Hyper Parameters

Varying the degree of non-linearity

\[ \rho = \text{ReLU}_k^L(x) \quad \text{where} \quad \text{ReLU}_k^L(x) = \begin{cases} \text{ReLU}(x(l)), & \text{if } l \leq k \\ x(l), & \text{otherwise} \end{cases} \]

"More non-linear" is better?

Varying the width \( K \)

- It permits to study a smaller model!

Global Contraction and Local Separation

Progressive contraction:

- Intra-class distances with depth

Cumulated variances with depth

Axis of PCA

We study: \[ \Sigma_k = \sum_{p \leq k} \sigma_p \]

Margin:

Cumulative distributions of:

\[(\{x^{(k)}_i - x^j\}_i)_{y(x^{(k)}_i) = y(x^j)} \quad \text{and} \quad (\{x^{(k)}_i - x^j\}_i)_{y(x^{(k)}_i) \neq y(x^j)} \]

Local Support Vectors: Exploring Regularity

- Estimating the intrinsic dimension of the classification boundary is hard (curse of dimensionality): we introduce local support vectors:
  \[ \Gamma_j = \{ x_j \mid y(x^{(k)}_i) = y(x^j) \} \]

- It permits to measure the regularity of the classification boundary at depth \( j \):
  \[ \Gamma^{j+1} = \{ x_j \in \Gamma^j \mid \text{card}(y(x_j) \neq y(x^{(k)}_i), l \leq k + 1) > \frac{k + 1}{2} \} \]

Conclusion

- Which mechanisms permit the dimensionality reduction to occur? Linearisation of complex variabilities?

- Theoretical guarantees are necessary to engineer and understand better deep networks.

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